Warm-up!

$y - y_1 = m(x - x_1)$

Unit 1 Day 5 (1.5)
- Even and Odd Functions
- Systems of Equations

Homework
- Even/Odd Functions + Systems of Equations Worksheet

First Quiz of The Year: Wednesday/Thursday (the class after next). It includes everything except inequalities.

1) Let f be the line passing through $(3,5)$ and $(9,2)$. Find $f(13)$.

$\frac{2-5}{9-3} = \frac{-3}{6} = -\frac{1}{2}$

$y - 2 = -\frac{1}{2}(x - 9)$

$y = -\frac{1}{2}(x-9) + 2$

$f(x) = -\frac{1}{2}(x-9) + 2$

$f(13) = -\frac{1}{2}(13-9) + 2 = -\frac{1}{2}(4) + 2 = 0$

$(13, f(13))$

$(3, 5)$

$\frac{f(13) - 5}{13 - 3} = -\frac{1}{2}$

$\frac{f(13) - 5}{10} = -0.5$

$f(13) - 5 = (10)(-0.5)$

$f(13) - 5 = -5$

$f(13) = 0$

1. Let h be a function passing through $(1,14)$ and $(5,23)$. Pretend h is a line, then find $h(8)$.

$\frac{23-14}{5-1} = \frac{9}{4}$

$y - y_1 = m(x - x_1)$

$y - 23 = \frac{9}{4}(x - 5)$

$h(x) = 23 + \frac{9}{4}(x-5)$

$h(8) = 23 + \frac{9}{4}(3) = 23 + \frac{27}{4} = 23 + 6.75$

$= 29.75$

$(8, h(8))$

$(5, 23)$

$\frac{h(8) - 23}{\cancel{8-5}\ 3} = \frac{9}{4}$

$h(8) - 23 = \frac{27}{4}$

$h(8) = 23 + \frac{27}{4}$

$h(8) = 29.75$

1. If $f(x)$ is an even function and $f(4) = -7$, find $f(-4)$.

$f(-4) = -7$

$f(-x) = f(x)$

1. If $g(x)$ is an odd function and $g(3) = -10$, find $g(-3)$.

$g(-3) = 10$

$f(x) = x^2$

Even and Odd Functions

Even Functions

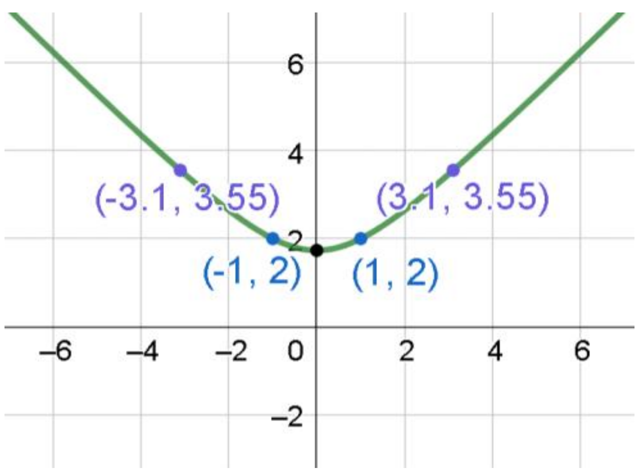
$$f(x) = \sqrt{x^2 + 3}$$

Numerically: Plugging in opposite x-values gives identical y-values

Graphically: y-axis symmetry

Function Composition: $f(-x) = f(x)$

Points: If (a, b) is on the graph, then $(-a, b)$ is also on the graph



Odd Functions

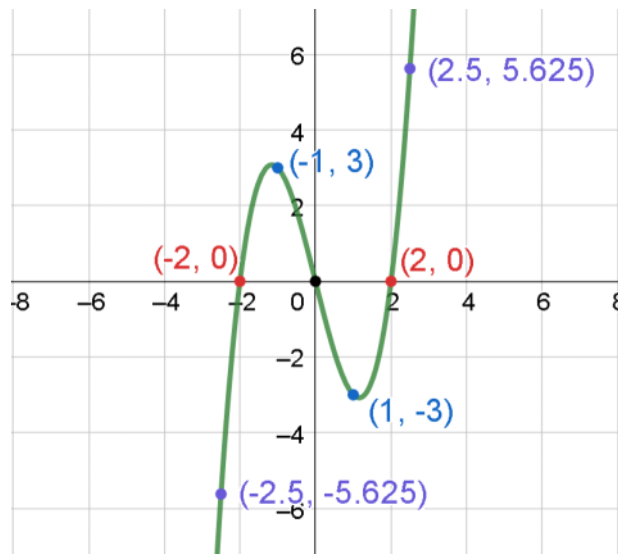
$$f(x) = x^3 - 4x$$

Numerically: Plugging in opposite x-values gives opposite y-values

Graphically: origin symmetry

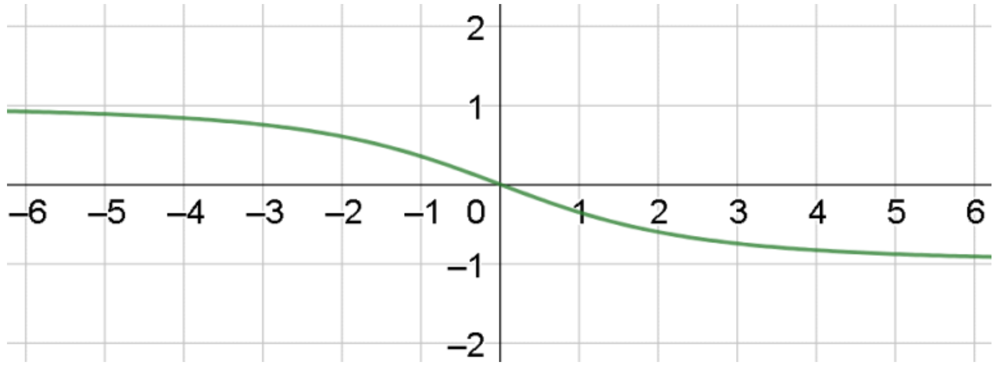
Function Composition: $f(-x) = -f(x)$

Points: If (a, b) is on the graph, then $(-a, -b)$ is also on the graph



Determine if the function is even, odd, or neither

A) $h(x) = \frac{-x}{\sqrt{7+x^2}}$

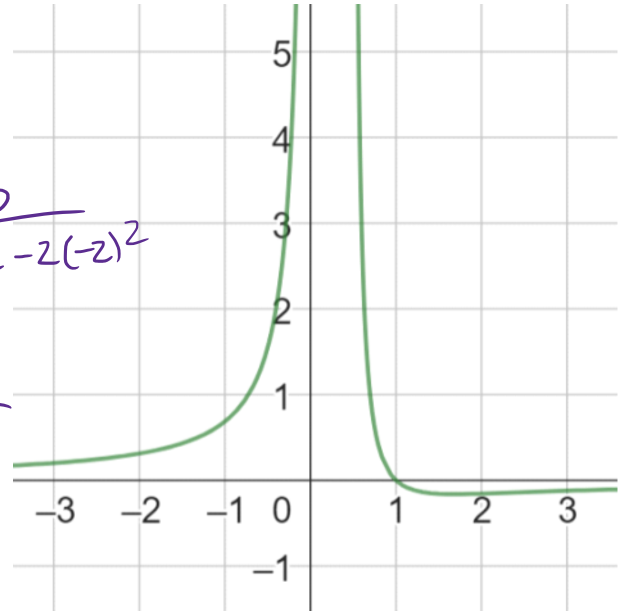


$h(1) = \frac{-1}{\sqrt{8}}$

$h(-1) = \frac{1}{\sqrt{8}}$

Determine if the function is even, odd, or neither

B) $f(x) = \frac{x-1}{x-2x^2}$



$f(2) = \frac{1}{2-2(2)^2}$

$\frac{1}{2-8}$

$f(2) = -\frac{1}{6}$

$f(-2) = \frac{-3}{-2-2(-2)^2}$

$\frac{-3}{-2-8}$

$\frac{-3}{-10}$

$f(-2) = \frac{3}{10}$

Determine if the function is even, odd, or neither

C) $g(x) = 2x|x^2 - 3|$

$g(1) = 2(1)|1-3|$

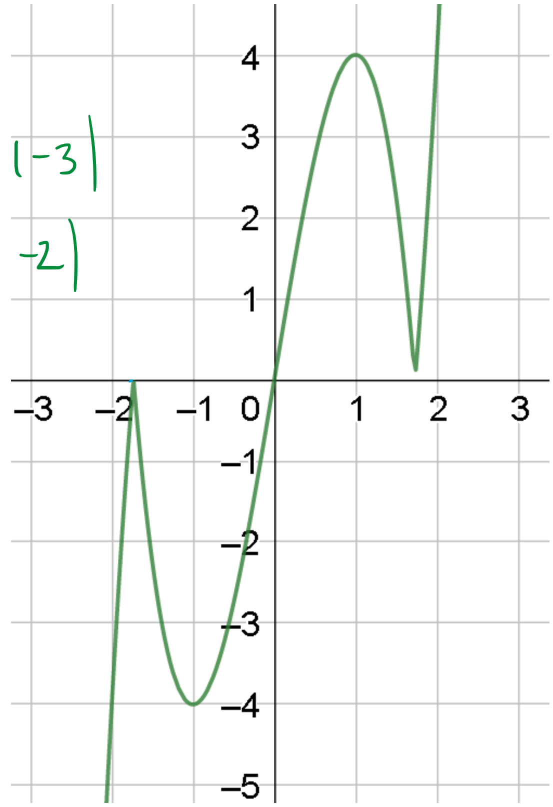
$2(1)|-2|$

$g(1) = 4$

$g(-1) = 2(-1)|1-3|$

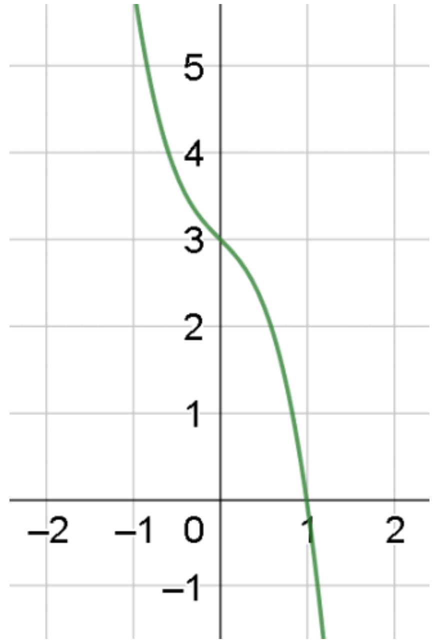
$2(-1)|-2|$

$g(-1) = -4$



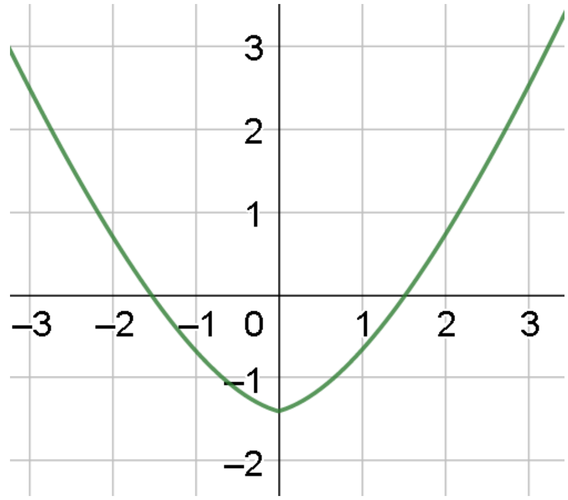
Determine if the function is even, odd, or neither

D) $f(x) = -2x^3 - x + 3$



Determine if the function is even, odd, or neither

E) $r(x) = \frac{3x^2-7}{|x|+5}$



Consequences of Being Even/Odd

Let $f(x)$ be an even function. What else do we know if…

A) $(-2,-3)$ is on the graph of $y = f(x)$

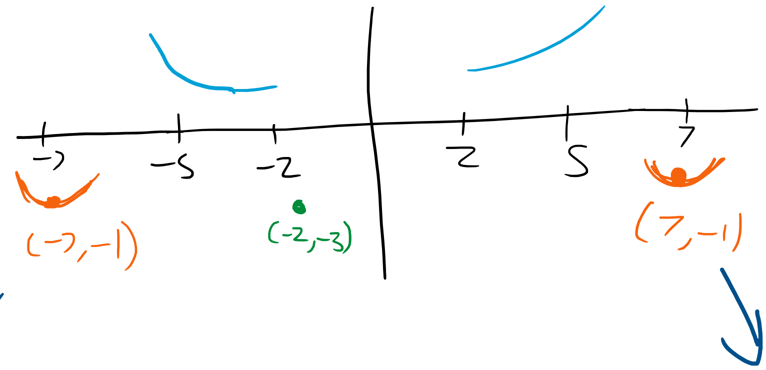
$(2,-3)$ is on the graph

B) $f(x)$ is increasing on the interval $(2,5)$

$f(x)$ is decreasing on $(-5,-2)$

C) $f(7) = -1$ is a relative minimum

$f(-7) = -1$ is a rel min

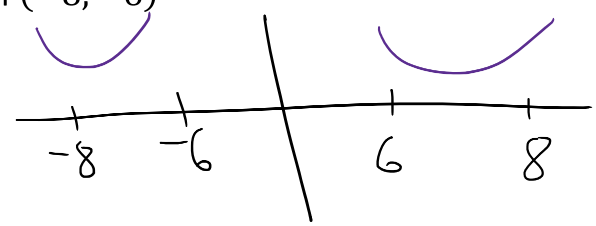


D) $\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = -\infty$

E) The rate of change of $f(x)$ is increasing on the interval $(-8,-6)$

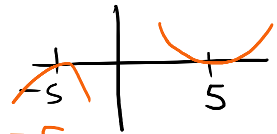
rate of change of $f(x)$ is increasing on $(6,8)$



Consequences of Being Even/Odd

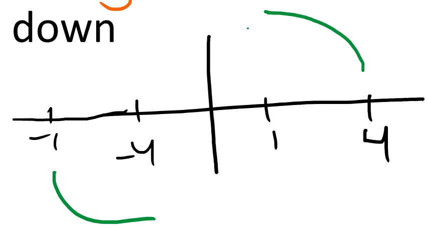
Let $g(x)$ be an odd function. What else do we know if…

F) The graph of g is tangent to the x-axis at $x = 5$



graph of g is tangent to x-axis at $x = -5$

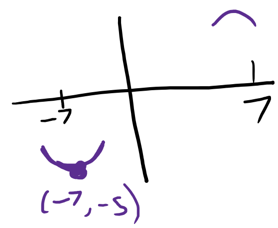
G) On the interval $(1,4)$, $g(x)$ is decreasing and concave down



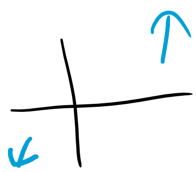
on $(-4,-1)$ $g(x)$ is dec and ccU

H) $g(-7) = -5$ is a relative minimum

$f(7) = 5$ is a rel max



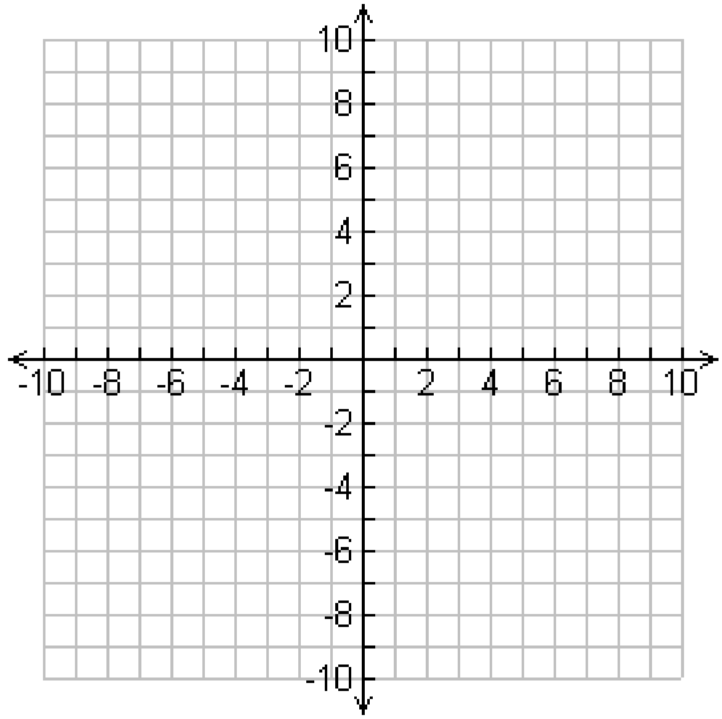
I) $\lim_{x\to-\infty} g(x) = -\infty$



$$\lim_{x\to\infty} g(x) = \infty$$

Sketch a polynomial $f(x)$ with the given characteristics

- $f(x)$ is even
- x is a factor of $f(x)$
- $\lim_{x \to \infty} f(x) = -\infty$
- $f(-3) = 2$ is a local maximum

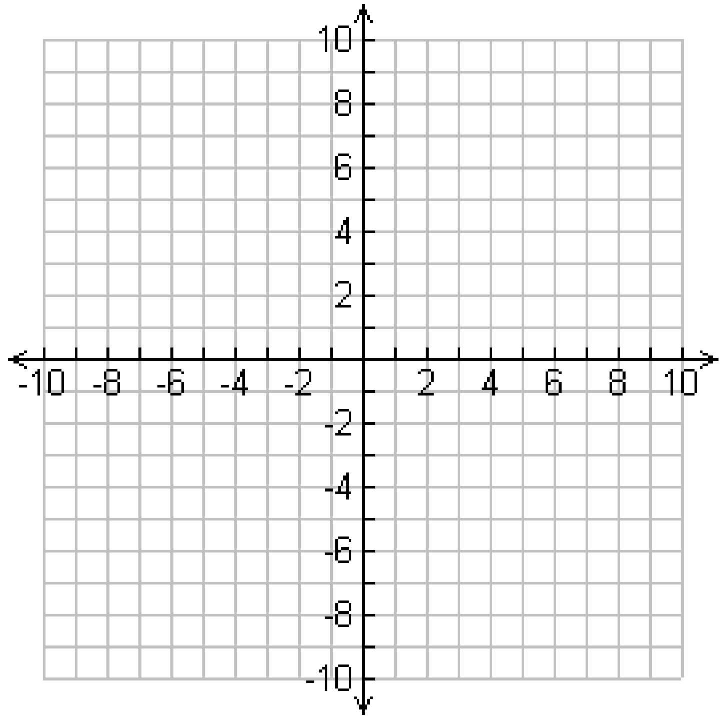


What is the minimum possible degree of this polynomial?

Is it possible this is a 6th degree polynomial? A 7th degree polynomial?

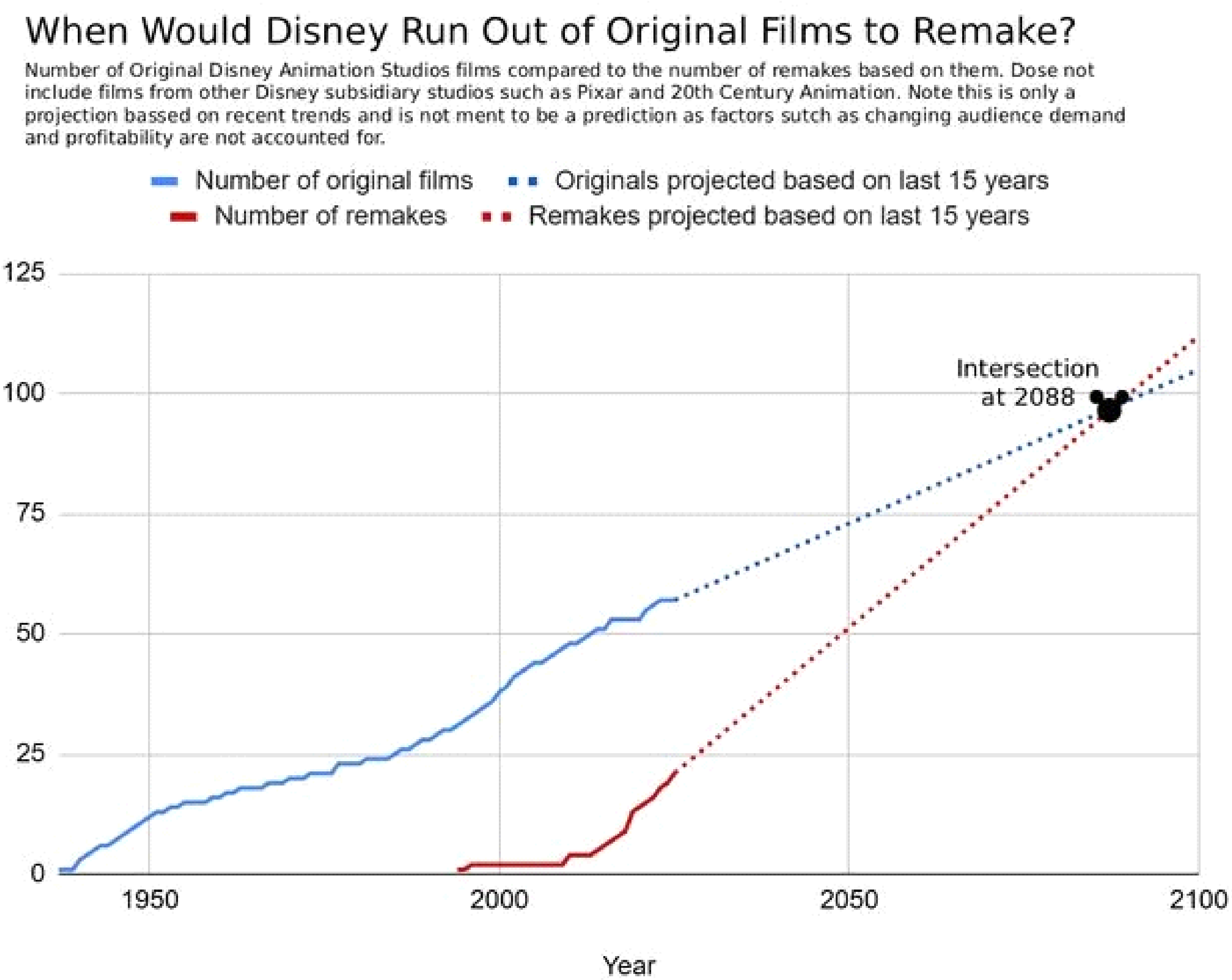
Sketch a polynomial $f(x)$ with the given characteristics

- $f(x)$ is odd
- $f(x)$ is cubic
- $2i$ is a zero of $f(x)$
- $\lim_{x \to -\infty} f(x) = -\infty$



Could $(0,8)$ be the y-intercept of this graph?

Linear Approximation



Consider a function $M(t)$ for which $M(2) = 8$ and $M(6) = 10$.

Find the average rate of change of M from $t = 2$ to $t = 6$.

$$\frac{M(6) - M(2)}{6-2} = \frac{10-8}{6-2} = \frac{2}{4} = \frac{1}{2} = 0.5$$

Use this average rate of change to predict $M(12)$

$$y - 10 = \frac{1}{2}(x-6)$$

$$y = \frac{1}{2}(x-6) + 10$$

$$M(t) \approx \frac{1}{2}(x-6) + 10$$

$$M(12) \approx \frac{1}{2}(12-6) + 10 = \frac{1}{2}(6) + 10 = 13$$

$(6, 10)$

$(12, M(12))$

$$\frac{M(12) - 10}{12-6} = \frac{1}{2}$$

$$\frac{M(12)-10}{6} = \frac{1}{2}$$

$$M(12) - 10 = 3 \rightarrow M(12) = 13$$

Use this average rate of change to predict $M(-5)$

$(-5, M(-5))$

$(2, 8)$

$$\frac{M(-5)-8}{-5-2} = \frac{1}{2}$$

$$\frac{M(-5)-8}{-7} = \frac{1}{2}$$

$$M(-5) - 8 = -\frac{7}{2}$$

$$M(-5) - 8 = -3.5$$

$$M(-5) = 4.5$$

Consider the function $R(t) = -t^2 - 7t + 5$

Find $R(-1)$ and $R(2)$

$R(-1) = 11$ $\qquad$ $R(2) = -13$

Find the average rate of change of R over the interval $-1 \leq t \leq 2$

$$\frac{R(2) - R(-1)}{2 - -1} = \frac{-13 - 11}{3} = -8$$

2.2
6.2
12.5
6.6
-9.8

Apply that average rate of change to estimate $R(1.6)$.

$(1.6, R(1.6))$
$(2, -13)$

$$\frac{R(1.6) - -13}{1.6 - 2} = -8$$

$$\frac{R(1.6) + 13}{-0.4} = -8$$

$$R(1.6) + 13 = 3.2$$

$$R(1.6) = -9.8$$

$(1.6, R(1.6))$
$(-1, 11)$

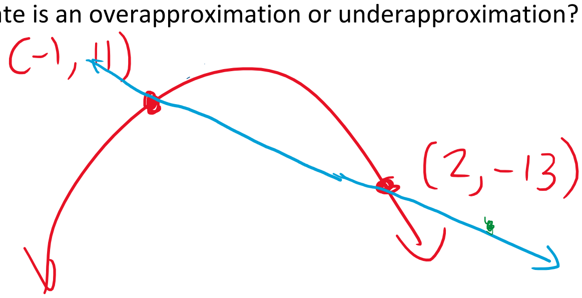
$$\frac{R(1.6) - 11}{1.6 - -1} = -8$$

$$\frac{R(1.6) - 11}{2.6} = -8$$

$$R(1.6) - 11 = -20.8$$

$$R(1.6) = -9.8$$

Without actually finding $R(1.6)$, can we predict whether our estimate is an overapproximation or underapproximation?



Justifying whether linear approximation overestimates or underestimates

Be sure to include:

- Concavity of the function you are approximating
- "secant line" to refer to estimates from the AROC
- t-values of secant line
- t-value(s) the question is about
- Relationship between location of secant line and the function you are approximating. Use the word "above" or the word "below" to compare them.

Highly encouraged first step: draw a rough sketch. Label it.

On any assessment this semester, this justification will be either fill in the blank or multiple choice. Second semester you will have to write the justification without any crutch.

Let $h(x) = 2x^2 + x - 5$

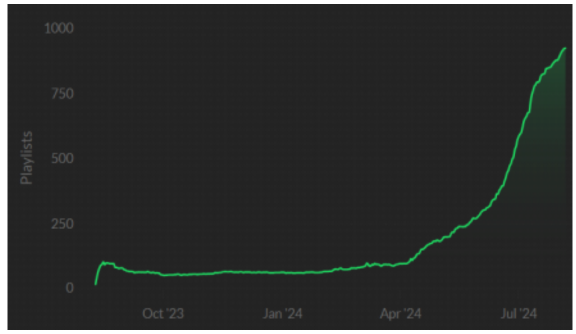
(i) Find the average rate of change of h over the interval $-2 \leq x \leq 4$

(ii) Use the average rate of change found in (i) to estimate $h(5.2)$

(iii) Let $L(x)$ be a linear function that estimates the value of $h(x)$ based on the average rate of change computer in (ii). For $-2 \leq x \leq 4$, which is greater, the actual value of $h(x)$ or the linear approximation? Use concavity to explain how you know.

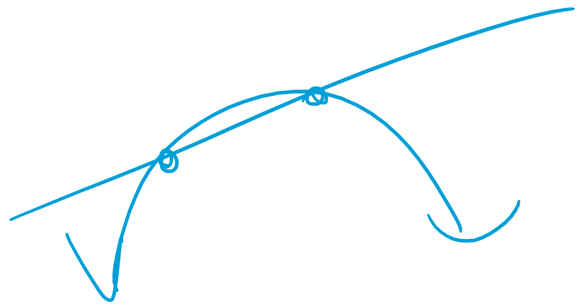
Chappell Roan's "HOT TO GO!" was released August 11, 2023. Three months after being released ($t = 3$), it could be found on 58 spotify playlists. Ten months after being released ($t = 10$), it could be found on 338 Spotify playlists. Let $N(t)$ be a function that includes both of the given data points to model how many Spotify playlists included "HOT TO GO!" t months after the song was released.

$N(3) = 58$
$N(10) = 338$



(i) Use the given data to find the average rate of change of $N(t)$, in playlists per month, from $t = 3$ to $t = 10$. Express your answer as a decimal approximation.

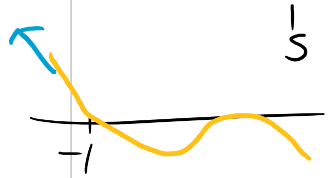
(ii) Use the average rate of change found in (i) to estimate the number of Spotify playlists that contained "HOT TO GO!" twelve months after the song's release.



(iii) Assume that $N(t)$ is concave up for $0 < t < 24$. Do you predict our estimate from part (ii) is an overestimate or underestimate of the actual number of playlists that included "HOT TO GO!" twelve months after the song's release? Justify.

6. The polynomial r is given by $r(x) = (2x^2 - 6x + 5)(2x^2 - 11x + 5)$. Which one of the following statements is true?
(A) r has four distinct real zeros
(B) r has two distinct real zeros and no non-real zeros
(C) r has no real zeros and two distinct non-real zeros
(D) r has two distinct real zeros and two distinct non-real zeros

$-5(x-5)(x-5)(x+1) = -5x^3 + \dots$



7. The polynomial f is given by $f(x) = -5(x-5)^2(x+1)$. Which one of the following must be true?
(A) Between $x = -1$ and $x = 5$ there must be an input value corresponding to a local maximum of f.
(B) Between $x = -1$ and $x = 5$ there must be an input value corresponding to a local minimum of f.
(C) At $x = 5$, f has a local minimum.
(D) As $x = -1$, f has a local maximum.

8. Construct a polynomial for which 1 is a zero of multiplicity 2. Make your polynomial have leading coefficient 1 and use the smallest possible degree for your polynomial. Write your polynomial in standard form.

$20 = A \cdot 5 \cdot 2 \cdot 1$
$f(x) = A(x+2)(x-1)(x-2)$

9. Construct a third-degree polynomial for which $f(-2) = 0$, $f(1) = 0$, $f(2) = 0$, and $f(3) = 20$.

$x = 3+2i$ $x = 3-2i$ $x = \frac{1}{2}$ (mult 2)

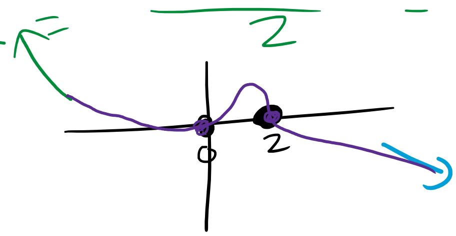
10. Find all zeros of $f(x) = (x^2 - 6x + 13)(4x^2 - 4x + 1)$. For each zero, determine the multiplicity.

$x^2 - 6x + 13 = 0$ $(2x-1)(2x-1)$

$x = \dfrac{6 \pm \sqrt{36-52}}{2} = \dfrac{6 \pm \sqrt{-16}}{2} = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

11. $f(x)$ is a polynomial with the following properties:
- The graph of $f(x)$ is tangent to the x-axis at $x = 5$
- $f(3) = 0$
- $\frac{2}{3} + i$ is a zero of f

What is the smallest possible degree of f?



12. $f(x)$ is a polynomial with the following properties:
- $(x - 2)$ is a factor of $f(x)$
- $f(0) = 0$
- As input values of $f(x)$ decrease without bound, the output values increase without bound.
- As input values of $f(x)$ increase without bound, the output values decrease without bound.

What is the smallest possible degree of f? $= 3$

$\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

Each of the following questions describes a polynomial function. However, each scenario is impossible. Give a reason why the proposed polynomial is impossible.

13. $f(1) = 0$, $f(4) = 0$, f has no relative extrema, $\lim_{x \to \infty} f(x) = -\infty$

14. All zeros of f have an even multiplicity, $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = \infty$, the graph of $f(x)$ has three x-intercepts

CBHS AP Precalculus 2025/26 - Unit 1, Day 4 HW

d. 2
e. (0,0)
f. (4,0)
2.
a. 4
b. 1

14. All zeros of f have an even multiplicity, $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$, the graph of $f(x)$ has three x-intercepts

   d. 2
   e. $(0,0)$
   f. $(4,0)$
2.
   a. 4
   b. 1
   c. 2
   d. 1
   e. None
   f. $(-2,0)$
3.
   a. 4
   b. 3
   c. 0
   d. 3
   e. $\left(\frac{3}{2},0\right)$ and $(3,0)$
   f. $(-3,0)$
4. $(0,-36)$
5. D
6. D
7. B
8. $f(x) = x^2 - 2x + 1$
9. $f(x) = 2(x+2)(x-1)(x-2)$ or $f(x) = 2x^3 - 2x^2 - 8x + 8$
10. $3 + 2i$ (multiplicity 1)
    $3 - 2i$ (multiplicity 1)
    $\frac{1}{2}$ (multiplicity 2)
11. 5 (AKA quintic)
12. 3
13. Between two real zeros of a polynomial, there must a relative extremum. If $f(1) = 0$ and $f(4) = 0$, somewhere between 1 and 4 is at least one x-value corresponding with a relative extremum.
14. The degree of a polynomial will always match the sum of the multiplicities of its zeros. If all the zeros are of an even multiplicity, the polynomial will be of an even degree. So, the left end behavior and right end behavior should match. This contradicts the limits given.

$x^2 - 6x + 13 = 0$

. $f(x)$ is a polynomial with the following properties:

- The graph of $f(x)$ is tangent to the x-axis at $x = 5$
- $f(3) = 0$
- $\frac{2}{3} + i$ is a zero of f

$x = \dfrac{6 \pm \sqrt{36 - 52}}{2}$

What is the smallest possible degree of f?

a zero

- $\frac{2}{3} + i$ is a zero of f

What is the smallest possible degree of f?

$\frac{2}{3} - i$ is a zero